

Metastability in spin polarised Fermi gases and quasiparticle decays

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Abstract. We investigate the metastability associated with the first order transition from normal to superfluid phases in the phase diagram of two-component polarised Fermi gases. We begin by detailing the dominant decay processes of single quasiparticles. Having determined the momentum thresholds of each process and calculated their rates, we apply this understanding to a Fermi sea of polarons by linking its metastability to the stability of individual polarons, and predicting a region of metastability for the normal partially polarised phase. In the limit of a single impurity, this region extends from the interaction strength at which a polarised phase of molecules becomes the groundstate, to the one at which the single quasiparticle groundstate changes character from polaronic to molecular. Our argument in terms of a Fermi sea of polarons naturally suggests their use as an experimental probe. We propose experiments to observe the threshold of the predicted region of metastability, the interaction strength at which the quasiparticle groundstate changes character, and the decay rate of polarons.

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1. Introduction

The experimental realisation of spin polarised ultracold Fermi gases has initiated a variety of new physics [1, 2, 3, 4, 5, 6, 7, 8, 9]. Of particular interest is the understanding of strongly interacting two-component Fermi gases at zero temperature. Two theoretical approaches have been used to shine light on this intriguing problem: the study of a single atom immersed in an ideal Fermi gas of atoms in a different spin state and, a Monte Carlo calculation at finite polarization. The Monte Carlo approach revealed the theoretical phase diagram of a homogeneous Fermi gas, as a function of polarisation $P = (N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow)$ and interaction strength [10], mapping out a phase separation of superfluid and normal phases. The single impurity approach examines the quasiparticles used as the building blocks to describe these phases. In the strongly imbalanced limit, a single \downarrow fermion immersed in a Fermi sea of \uparrow fermions, the spin impurity atom becomes either a fermionic (polaron) or bosonic (molecule) quasiparticle. Complementary wave functions for each quasiparticle ([11, 12] and [13, 14, 15], respectively) provide groundstate energies and effective masses. Previous studies showed that the critical interaction strength at which the groundstate of a single impurity at zero momentum switches from the fermionic to bosonic branch [16] occurs at $1/(k_{F\uparrow}a)_c \sim 0.88$, with $k_{F\uparrow} = (6\pi^2 n_\uparrow)^{1/3}$ the Fermi momentum of a non-interacting Fermi sea of \uparrow atoms with density n_\uparrow , and a the scattering length parametrizing the interaction strength between \uparrow and \downarrow atoms. This value is higher than the interaction strength $1/k_{F\uparrow}a \sim 0.73$ at which a superfluid phase emerges in the limit of full polarisation [10]. An important conclusion made from this is that no measurement made in the groundstate will allow us to see the point at $1/(k_{F\uparrow}a)_c$. As we will show later the use of metastable and out of equilibrium processes overcomes this problem.

Firstly, we determine the momentum thresholds for the decay of single impurities with finite momentum. We then consider the metastability associated with the normal to superfluid first order phase transition in the thermodynamic phase diagram. Indeed by applying the understanding of single quasiparticle decay to a Fermi sea of polarons, we predict that there exists a region where such a phase is metastable. We propose the use of a Fermi sea of polarons as an experimental probe to determine the threshold of the region. As this threshold goes to zero at $1/(k_{F\uparrow}a)_c$, we can observe this point for the first time. Beyond this threshold the decay of polarons into molecules may lead to a mixture of molecules and polarons, which suggests a possible way of measuring the molecule-polaron scattering length. In this way, we hope to open up a new regime of metastable physics in Fermi gases for experimental exploration. Finally, we calculate the decay rates for each process, within the key regions of interaction strength and momenta, to determine the fate of the quasiparticles. The presence of a Fermi sea of polarons would again be instrumental to measure the various decay rates.

2. Background

Our starting point is an understanding of the single quasiparticle groundstate as a function of $1/k_{F\uparrow}a$ going from unitarity ($1/k_{F\uparrow}a \rightarrow 0$) to the “BEC” limit ($1/k_{F\uparrow}a \rightarrow +\infty$) (see figure 1). At $1/(k_{F\uparrow}a)_c \sim 0.88$, the critical point, the zero momentum energies of the polaron and molecule cross. For smaller values of $1/k_{F\uparrow}a$ the polaron is the groundstate of the system, and for larger values the molecule is the groundstate. In figure 1 and throughout this paper we use the polaron energy

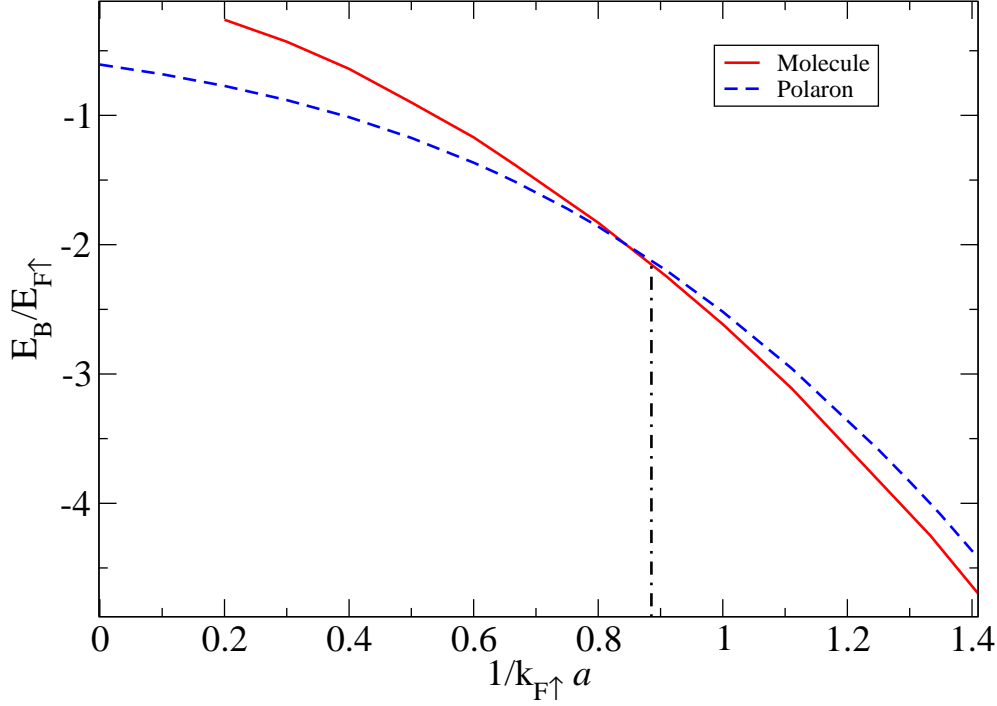


Figure 1. Molecule and polaron zero momentum energies as a function of interaction strength on the “BEC” side. Red line: Molecule. Blue dashed line: Polaron. The dot-dashed line marks the critical interaction strength $1/(k_{F\uparrow}a)_c \sim 0.88$. To the left of it the polaron energy is lower, and to the right it is the molecule that becomes the groundstate.

calculated using the wave function proposed by Chevy [11]. The energy in the BCS limit tends to $E_{Pol} = 4\pi a n_{\uparrow}/m$, the mean-field interaction energy, and in the BEC limit tends to $E_{Pol} = -\frac{1}{ma^2} - \frac{\epsilon_{F\uparrow}}{2}$. Here and in the following, we take $\hbar = 1$. The effective mass of a polaron (m_{Pol}^*) obtained from [15] becomes just the bare mass in the BCS limit and diverges at $1/k_{F\uparrow}a \sim 1.17$. For the molecule, energies and effective masses are obtained from [13, 14, 15]. In the BEC limit the energy tends to $E_{Mol} = -\frac{1}{ma^2} - \epsilon_{F\uparrow}$. The effective mass equals the bare molecule mass $m_{Mol}^* = 2m$ in that limit, while approaching unitarity it grows, and it diverges at $1/k_{F\uparrow}a \sim 0.55$. These masses and energies for both polarons and molecules have been found to be very accurate by comparison with Quantum Monte Carlo calculations [10, 16].

3. Decay processes and thresholds

We begin by considering the decays that the polarons and molecules can undergo when they are not in the groundstate (e.g. at finite momentum). The decay process of a quasiparticle can only occur when it satisfies energy and momentum conservation. This is generally only possible when the initial quasiparticle momentum p is above a threshold momentum P_{Th} .

3.1. Polaron decay

For the polaron at zero temperature, the decay processes we consider are:

- A. $Polaron \rightarrow Molecule + hole$
- B. $Polaron \rightarrow Molecule + 2\ holes + particle$

A finite momentum polaron also has a finite relaxation time as described in [17], which schematically reads

- C. $Polaron(p) \rightarrow Polaron(p' < p) + hole + particle$

The three-body decay of a polaron into a molecule, two holes and a particle has been considered in [18] in the special case $\mathbf{p} = 0$. Generalizing this calculation to non-zero momentum, we find that process B can occur at any momentum when $\Delta E = E_{Pol} - E_{Mol} > 0$, as expected, and only for

$$p > P_{Th}^B = \sqrt{-2m_{Pol}^* \Delta E} \quad (1)$$

when $\Delta E < 0$. Here E_{Pol} and E_{Mol} are the energies of a zero-momentum polaron and molecule at the given value of interaction strength.

For process A, the two-body decay of a polaron with momentum \mathbf{p} into a molecule and a hole, conservation of energy and momentum require the following equality to be verified:

$$E_{Pol} + \frac{p^2}{2m^*} + \xi_q = E_{Mol} + \frac{(\mathbf{p} + \mathbf{q})^2}{2M^*}, \quad (2)$$

where $\xi_q = \frac{q^2}{2m} - \epsilon_{F\uparrow}$ is the kinetic energy of a majority particle measured with respect to the Fermi surface. The minimum momentum P_{Th}^A at which process A is allowed is found by setting the hole at the Fermi surface, and by taking its momentum to be anti-parallel to the one of the polaron:

$$\frac{P_{Th}^A{}^2}{2m_{Pol}^*} + E_{Pol} = \frac{(P_{Th}^A - p_F)^2}{2m_{Mol}^*} + E_{Mol}. \quad (3)$$

The processes including more particle-hole pairs (e.g. $Molecule + 3\ holes + 2\ particles$) have the same energy threshold as the state resulting from process B, but lower rates so we do not consider them here.

In conclusion, polarons with non-zero momentum are stable towards A/B decay for momenta smaller than $P_{Th}^{A/B}$.

Figure 2 shows our results for the polaron decay thresholds, as given by (1) and (3). In the region below $1/(k_{F\uparrow}a)_c$, where the zero momentum polaron is the groundstate of the system, a polaron at finite momentum can nevertheless be unstable to decay processes A, B, and C. The solid blue line gives the momentum threshold P_{Th}^B for the polaron to decay via process B. On this line, decay results in a zero momentum molecule for $1/(k_{F\uparrow}a) < 1/(k_{F\uparrow}a)_c$. Finite momentum molecules also result from process B above the solid blue line. The blue dashed line gives the momentum threshold P_{Th}^A for process A, which generally creates a molecule at finite momentum even on the threshold. Decay processes including more particle-hole pairs all set in at momenta above the solid blue line, i.e. for $p > P_{Th}^B$. For $1/(k_{F\uparrow}a) > 1/(k_{F\uparrow}a)_c$, a polaron at zero momentum is unstable to a process B decay into a molecule at finite momentum whereas process A continues to affect only higher momentum polarons. Polarons with any finite momentum are unstable to momentum relaxation (process C) on both sides of $1/(k_{F\uparrow}a)_c$. The momentum thresholds for processes A and B become equal where the molecule's effective mass diverges ($1/k_{F\uparrow}a \sim 0.55$).

3.2. Molecule decay

The stability of a molecule is calculated in a similar way. The decay channels we consider are:

- \tilde{A} . *Molecule* \rightarrow *Polaron* + *particle*
- \tilde{B} . *Molecule* \rightarrow *Polaron* + 2 *particles* + *hole*
- \tilde{C} . *Molecule* (p) \rightarrow *Molecule* ($p' < p$) + *hole* + *particle*

For $1/k_F a < 1/(k_{F\uparrow} a)_c$, a zero momentum molecule decays via process \tilde{B} to a polaron with finite momentum, and via process \tilde{A} at higher momentum. For $1/k_F a > 1/(k_{F\uparrow} a)_c$, process \tilde{B} preceeds process \tilde{A} with increasing momentum until where the polaron's effective mass diverges. In analogy with the polaron decay considered above, the momentum thresholds for the molecule are determined by energy and momentum conservation. They are,

$$\frac{P_{Th}^{\tilde{A}}^2}{2m_{Mol}^*} + E_{Mol} = \frac{(P_{Th}^{\tilde{A}} - p_F)^2}{2m_{Pol}^*} + E_{Pol}, \quad (4)$$

$$P_{Th}^{\tilde{B}} = \sqrt{2m_{Mol}^* \Delta E} \quad \text{for} \quad \frac{1}{k_{F\uparrow} a} > \frac{1}{(k_{F\uparrow} a)_c}, \quad (5)$$

where $P_{Th}^{\tilde{B}} = 0$ for $1/k_{F\uparrow} a \leq 1/(k_{F\uparrow} a)_c$. Again, the threshold momentum for process \tilde{A} is found when the majority particle is on the Fermi surface.

Our results for the threshold momenta for polaron and molecule decay, as given by (1), (3), (4) and, (5) are shown in figure 2.

4. Metastability of polaron gas

Thus far we have analysed the decay processes of single quasiparticles. The single quasiparticle problem is a limiting case of the spin imbalanced Fermi gas. The groundstate phase diagram of a spin imbalanced Fermi gas, with polarisation versus interaction strength, has been calculated in [10] using a Monte Carlo approach. In this calculation, the \downarrow atom concentration is kept finite for a macroscopic system even in the $P \rightarrow 1$ limit. We now examine how the stability analysis above is connected to this equilibrium phase diagram.

Our particular consideration is the normal to superfluid first order phase transition predicted by the Monte Carlo calculations [10]. This transition from a partially polarised normal phase to a state with separated superfluid and normal phases is represented by the dashed green line in figure 3. To make this line in figure 3, we have assumed that the polarons form a weakly interacting Fermi sea, so that we can convert the critical density at which phase separation occurs, into a critical Fermi momentum $p_F^{Pol}(1/k_{F\uparrow} a)$ for the polarons. In the phase separated state, the Monte Carlo calculation includes the interactions between the molecules in the condensate. In contrast, in the single \downarrow atom calculation there is at most one molecule and therefore no condensate.

If we assume that the relevant processes in which the polarons can be converted into molecules, within experimentally realistic timescales, are only the ones considered in section 3.1 and that these single impurity processes can be used to analyse the stability of the Fermi sea of polarons (neglecting for instance multiple polaron decay),

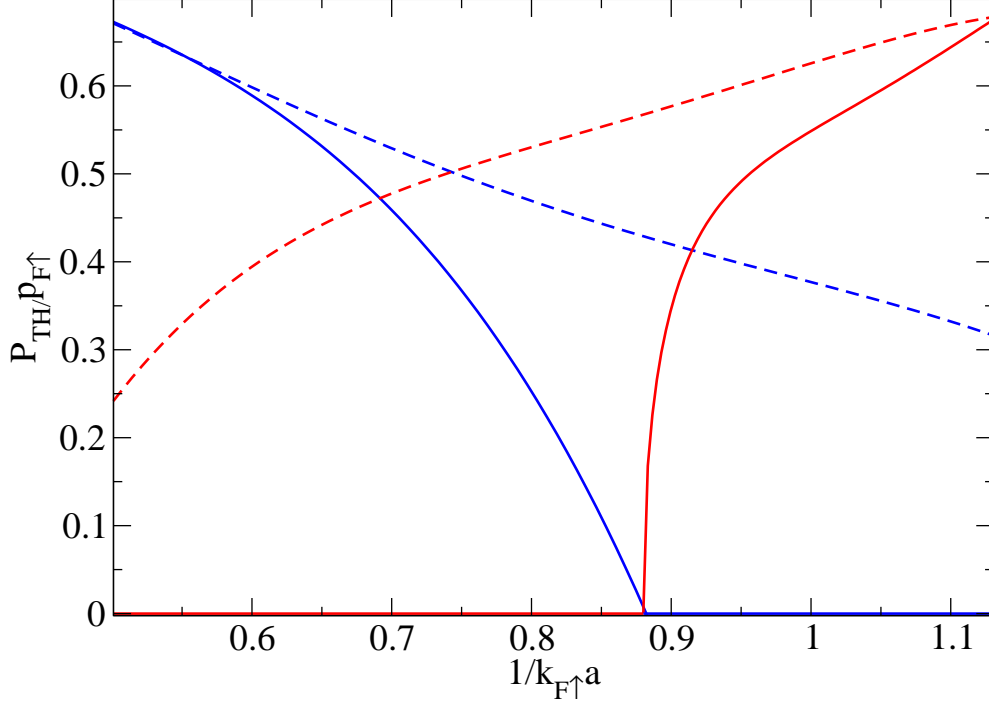


Figure 2. Momentum thresholds for various decay processes for both polarons and molecules about the critical point. Blue solid line: *Polaron* \rightarrow *Molecule* + 2 holes + particle; Blue dashed line: *Polaron* \rightarrow *Molecule* + hole; Red solid line: *Molecule* \rightarrow *Polaron* + 2 particles + hole; Red dashed line: *Molecule* \rightarrow *Polaron* + particle. The groundstate of the system changes from being a polaron to a molecule at $1/k_{F\uparrow}a \sim 0.88$, beyond which even a polaron at $p = 0$ can decay via process B; otherwise, only polarons at $p > 0$ are unstable in this way. Process A is relevant at higher polaron momentum and generally results in a finite momentum molecule even on the threshold. The process A and B thresholds meet when the molecule's (polaron's) effective mass diverges at $1/k_{F\uparrow}a \sim 0.55$ (1.17). Note that for polarons at large p ($\sim p_{F\uparrow}$), momentum relaxation processes are very strong so that such a polaron is no longer a well-defined quasiparticle.

this gives rise to a region of *metastability* in the phase diagram. If a Fermi sea of polarons prepared in the groundstate (below the green dashed line) is adiabatically taken above the dashed green line by increasing $1/k_Fa$, we expect it to persist as a metastable state. Even though the density of the \downarrow atoms is so high that the true groundstate is a phase separated state, the state is stable since there are no polarons with large enough momentum to decay to a molecule via process B. The decay of polarons to molecules sets in only when $1/k_Fa$ is increased further so that the Fermi momentum of the polarons is larger than the momentum threshold for process B ($p_F^{ol} = P_{Th}^B$); we ignore here molecule-polaron interactions (see below). This momentum is given by the solid blue line in figure 3. At this point, the highest momentum polarons will decay into molecules.

This motivates the following proposal to use a Fermi sea of polarons as an experimental probe to explore the metastable region and observe molecule formation through process B. Due to the Pauli exclusion principle, the momentum relaxation

process (C) is suppressed allowing for an analysis of the other decay processes. Let us suppose for the moment that the molecules into which polarons can decay do not interact with the polarons. Then, to determine the shape of P_{Th}^B , we can prepare a polaron Fermi sea with a Fermi momentum smaller than the dashed green line in figure 3 at unitarity, for example. Then we increase $1/k_{F\uparrow}a$ adiabatically. The system will cross the first order phase transition line, but since it is metastable (as discussed above), it will remain a Fermi sea of polarons rather than form a condensate of molecules. As we continue beyond the phase transition, the Fermi momentum will become equal to the momentum threshold for process B ($p_F^{Pol} = P_{Th}^B$). Increasing $1/k_{F\uparrow}a$ infinitesimally beyond this point will now lead to the decay of polarons at p_F^{Pol} . The polarons on the Fermi surface decay into molecules at zero momentum. The appearance of these molecules can then be detected experimentally as the tell-tale sign of the solid blue line P_{Th}^B . The experiment can be repeated using an appropriate p_F^{Pol} to find P_{Th}^B at different interaction strengths, making sure that only a small number of molecules are created each time (so as to be able to ignore effects beyond the threshold, e.g. molecule-molecule interactions) but a large enough number to be observable. The size of the polaron Fermi sea needed will decrease as P_{Th}^B decreases, so that the value $1/(k_{F\uparrow}a)_c$ is found in the limit of a single polaron.

If we now take into account molecule-polaron interactions the value of p_F^{Pol} at which molecules are first formed will change. If we consider one molecule in the final state, the (unknown) molecule-polaron interaction changes the energy by $\Delta E = g_{PM}n_{Pol}$ (assuming a mean-field approximation). This will lead to a positive or negative shift of the threshold curve of production of molecules (see dot-dashed blue lines in figure 3). This shift tends to zero at $1/(k_{F\uparrow}a)_c$ where $n_{Pol} \rightarrow 0$ and where the threshold curve meets P_{Th}^B . One could in principle use the difference between the experimentally observed curve and P_{Th}^B (which is known theoretically) to determine this shift and so the molecule-polaron scattering length. Note that we have ignored the effects of the polaron-polaron interaction, which are known to be small.

We have therefore established that the region between the dashed green and dot-dashed blue lines in figure 3 represents a metastable phase consisting of a Fermi sea of polarons. Such a metastable phase is characteristic of a first order phase transition. It is important to note that the state may well be metastable beyond the dot-dashed blue line since an analysis of its stability in that region would require us to take into account the presence of a finite quantity of molecules. We also raise the intriguing possibility of a final state containing the remaining Fermi sea of polarons coexisting with a condensate of molecules, within a background of \uparrow particles.

5. Single quasiparticle decay rates and experimental observability

5.1. Decay rates calculation

The phase diagram is animated by considering the rates of each decay process. The decay rates of zero momentum polarons and molecules via process B are presented in [18]. The decay rate of a polaron with momentum p through process A is given by the imaginary part of the on-shell polaron self-energy Σ_{Pol}^A shown in figure 4a, i.e. $\Gamma_{Pol}^A(p) = -\text{Im}\Sigma_{Pol}^A(p, E_{Pol} + \frac{p^2}{2m_{Pol}^*})$. At zero temperature, one obtains

$$\Gamma_{Pol}^{(A)}(p) = -\pi Z_{Mol} g^2 \int_{q < k_{F\uparrow}} \delta \left(\Delta E + \frac{p^2}{2m_{Pol}^*} + \xi_q - \frac{(p+q)^2}{2m_{Mol}^*} \right), \quad (6)$$

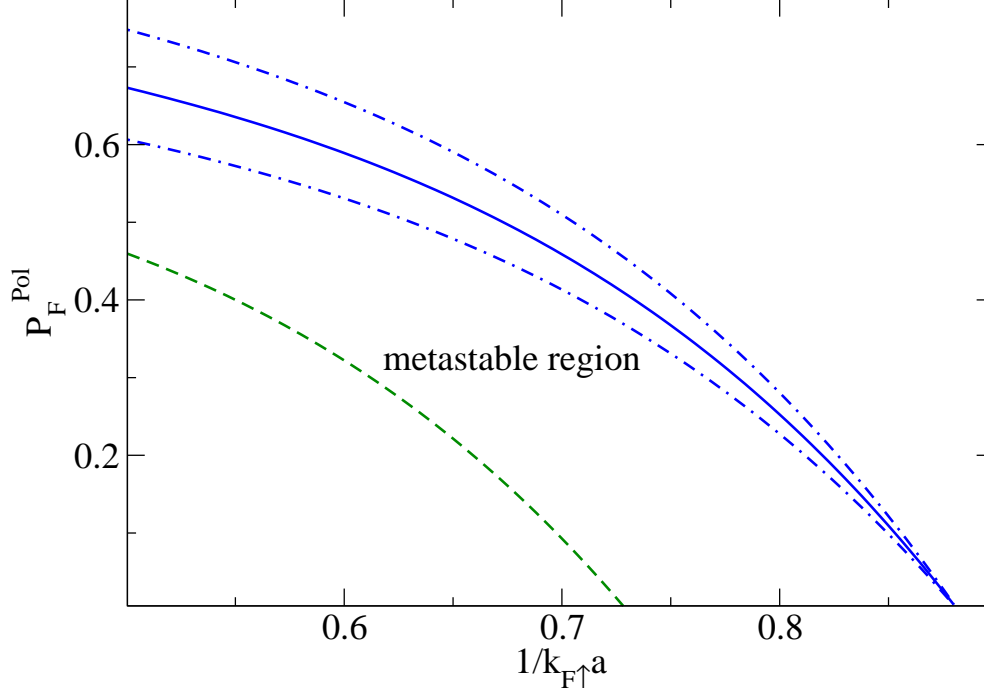


Figure 3. Stability of Fermi sea of polarons: The dashed green line shows the first order transition from normal to phases including a superfluid (Fig. 4 in [10]). The solid blue line is the Fermi momentum of a Fermi sea of polarons equal to the momentum threshold for process B ($p_F^{Pol} = P_{Th}^B$). The dot-dashed blue lines represent how the solid blue line might be shifted due to molecule-polaron interactions.

which is a simple Golden rule expression. Here, $g = -\sqrt{\frac{2\pi}{m_r^2 a}}$ is the atom-molecule coupling in vacuum [18], and m_r is the atom-molecule reduced mass. To derive this equation, we have used a pole expansion of the molecule propagator with quasiparticle residue Z_{Mol} . The decay rate may be calculated analytically. Assuming for simplicity $m_{Mol}^* = 2m_\uparrow$ and $m_{Pol}^* = m_\uparrow$, we obtain

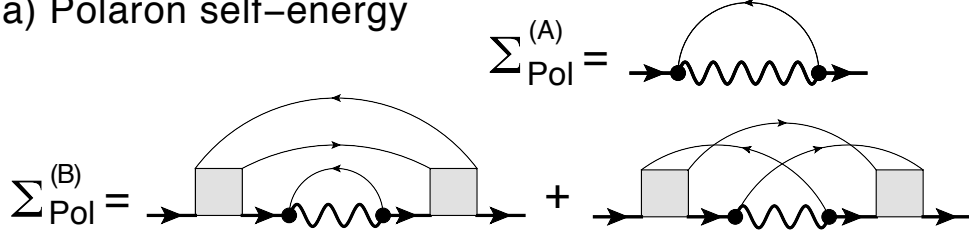
$$\Gamma_{Pol}^A(p) = Z_{Mol}\epsilon_{F\uparrow}\frac{4}{k_{F\uparrow}a}\frac{(p - P_{Th}^A)(P_{Th}^A + 2k_{F\uparrow} - p)}{pk_{F\uparrow}}, \quad (7)$$

for $P_{Th}^A < p < P_{Th}^A + 2k_{F\uparrow}$ and zero otherwise. The momentum threshold for this process is $P_{Th}^A = k_{F\uparrow}\left[\sqrt{2\left(1 - \frac{\Delta E}{\epsilon_{F\uparrow}}\right)} - 1\right]$. Note that $P_{Th}^A > 0$ even when $\Delta E > 0$, and that one has $\frac{\Delta E}{\epsilon_{F\uparrow}} < 0.5$ for all scattering lengths [13, 16].

The decay rate of a molecule with momentum p via process A, the creation of a polaron and a majority particle, is given by the imaginary part of the molecule self-energy Σ_{Mol}^A depicted in figure 4b. A calculation analogous to the polaron case considered above yields,

$$\Gamma_{Mol}^A(p) = Z_{Pol}\epsilon_{F\uparrow}\frac{2}{k_{F\uparrow}a}\frac{(p - P_{Th}^{\bar{A}})(p - P_{Th}^{\bar{A}} + 4k_{F\uparrow})}{pk_{F\uparrow}}, \quad (8)$$

(a) Polaron self-energy



(b) Molecule self-energy

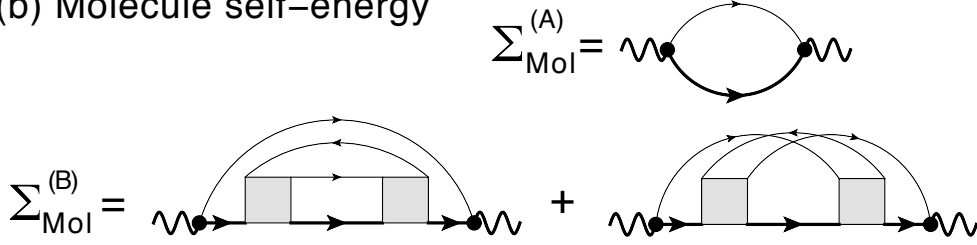


Figure 4. Diagrams showing the decay processes A and B for (a) polarons and (b) molecules. The thin lines represent majority atoms, wavy lines a molecule, and thick lines a polaron. The polaron-molecule matrix element g is represented by a thick dot and the square represents off-resonant scattering between a polaron and the majority atoms.

with Z_{Pol} the quasiparticle residue for the polaron. The threshold momentum for this process is $P_{Th}^A = k_{F\uparrow} \left[2 - \sqrt{2 \left(1 - \frac{\Delta E}{\epsilon_{F\uparrow}} \right)} \right]$ with $\Gamma_{Mol}^A = 0$ for $p < P_{Th}^A$. Note that in this case, as opposed to the polaron case, there is no maximum momentum for the molecule above which there is no decay via process A. This is because for large momentum, the molecule can always dispose its energy and momentum to a majority particle, whereas the polaron has to dispose it into a majority hole within the Fermi sea.

The corresponding self-energy for process B, Σ_{Pol}^B is shown in figure 4a. Generalising the calculations in [18] to non-zero momentum p gives

$$\Gamma_{\text{Pol}}^B(p) \propto Z_{\text{Mol}} \epsilon_{F\uparrow} \left[\frac{\left(\Delta E + \frac{p^2}{2m_{\text{Pol}}^*} \right)}{\epsilon_{F\uparrow}} \right]^{\frac{9}{2}}. \quad (9)$$

The analysis in [18] showed that the proportionality constant in this expression is of order unity.

Likewise, the decay rate of a molecule via process B can be obtained by generalising the calculations in [18] to a finite momentum p . We obtain

$$\Gamma_{\text{Mol}}^B(p) \propto Z_{\text{Pol}} \epsilon_{F\uparrow} \left[\frac{\left(-\Delta E + \frac{p^2}{2m_{\text{Mol}}^*} \right)}{\epsilon_{F\uparrow}} \right]^{\frac{9}{2}}, \quad (10)$$

where the constant of proportionality is, again, of order unity. P_{Th}^B ($P_{Th}^{\tilde{B}}$) is the point at which Γ_{Pol}^B (Γ_{Mol}^B) equals zero.

For a finite momentum, the polaron can scatter off the majority particles giving rise to momentum relaxation with a rate $\frac{1}{\tau_{Pol}}$ (process C). The high velocity regime $k_{F\downarrow} \ll m_{\downarrow}^* v \ll k_{F\uparrow}$ analysed in [17] determines the rate of relaxation of a single impurity. If we consider only a single impurity, spin statistics becomes redundant, allowing the same calculation to be used for the molecule. The momentum relaxation rate, at $T = 0$ for molecule or polaron is then,

$$\frac{1}{\tau_{Mol/Pol}} = \frac{2\pi}{35} |\gamma|^2 \frac{m_{Mol/Pol}^{*3} v^4}{k_{F\uparrow}^2}, \quad (11)$$

where γ is determined by the scattering amplitude $U = \frac{\partial \mu_{Mol/Pol}}{\partial n_{\uparrow}} = \frac{2\pi^2}{m_{\uparrow} k_{F\uparrow}} \gamma$. At unitarity, we have $\mu_{Pol} = -\alpha \epsilon_{F\uparrow}$ with $\alpha \simeq 0.6$ [16]. One can also use $\mu_{Mol} = -\frac{1}{ma^2} + \frac{3\pi\tilde{a}}{m} n_{\uparrow}$, valid for $\frac{1}{k_{F\uparrow} a} \gtrsim 0.7$ [16] (where $\tilde{a} \simeq 1.18a$) to find the molecule momentum relaxation rate in terms of the interaction strength,

$$\frac{1}{\tau_{Mol}} = \frac{9}{70\pi} (k_{F\uparrow} \tilde{a})^2 \frac{m_{Mol}^{*3} v^4}{k_{F\uparrow}^2}. \quad (12)$$

The high power of the velocity v and effective mass m^* indicate impurities at large momenta $p \sim p_{F\uparrow}$ are no longer well-defined quasiparticles.

In figure 5, we plot the decay rate of polarons and molecules via processes A and B as a function of momentum p for various ΔE . We see that once $p > P_{Th}^A$ and process A sets in, it quickly dominates over B. In fact, once active, process A dominates over B by several orders of magnitude since the ratio is, to lowest order $\frac{\Gamma_A}{\Gamma_B} \propto \left(\frac{p - P_{Th}^A}{p_{F\uparrow}} \right) \left(\frac{p_{F\uparrow}}{P_{Th}^A} \right)^{10}$. This is expected since process A is a 2-body process, whereas process B is a 3-body process. On the other hand, for $p < P_{Th}^A$ process B of course dominates as process A is not allowed. However, since the typical time scale for process B is very long, of order 10-100ms, the momentum relaxation of the polaron via process C is in general much faster. This justifies our proposed experiment to determine P_{Th}^B using a Fermi sea of polarons in equilibrium, to suppress process C.

5.2. Decay rates experiment

The polaron Fermi sea can also be used to measure the rate of process B. The initial state is a polaron Fermi sea prepared in the groundstate (i.e. below the dashed green line in figure 3). We then instantaneously increase the interaction strength so that the some fraction of the polarons have a momentum $p > P_{Th}^B$ and so are then susceptible to decay via process B. Alternatively, by increasing the interaction strength to above $1/(k_{F\uparrow} a)_c$, every polaron has a momentum $p > P_{Th}^B$, and so the whole Fermi sea is susceptible to decay via process B. The molecules resulting from this decay are unable to decay via process \tilde{A} or \tilde{B} , however they lose momentum via process \tilde{C} . The resultant state is therefore expected to be a condensate of molecules. A measurement of the initial growth rate of the number of molecules or the loss rate of polarons determines the rate of process B for polarons, averaged over the momenta of the polarons decaying, at a given $1/k_{F\uparrow} a$.

Similarly, the rate of process A can be measured, with some fraction of the polarons having a momentum $p > P_{Th}^A$. In this case, the Fermi sea of polarons will decay via both processes A and B. The two processes can be distinguished since

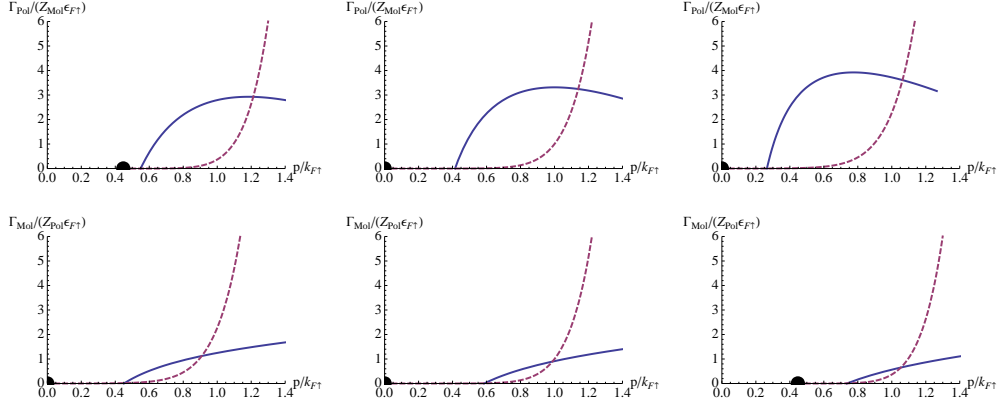


Figure 5. Decay rates of finite momentum polarons (top) and molecules (bottom) for processes A (continuous line) and B (dashed line). From left to right, we have $\Delta E/\epsilon_{F\uparrow} = -0.2, 0, 0.2$. The thick dot indicates the threshold momentum for process B, P_{Th}^B . Process A, above its threshold dominates over B for $p < p_{F\uparrow}$.

process A has a significantly faster rate and typically results in molecules at finite momentum.

6. Conclusions

In this paper, we have shown how quasiparticles in a two-component Fermi gas behave at finite momenta in the limit of extreme imbalance. By considering energy and momentum conservation of single impurities, we determined the momentum thresholds beyond which the quasiparticles are susceptible to the most significant decay processes, and we calculated the rates of each. Using this and assuming we can use single quasiparticle decay processes to describe the decay of a Fermi sea of polaron, we have identified a region of metastability for the partially polarised normal phase about the normal to superfluid first order transition. We then described how the Fermi sea can be used as an experimental probe to observe the metastable region, determine the crossing point of single impurity energies $1/(k_{F\uparrow}a)_c$ for the first time and measure the decay rate of process A and B. The phase diagram we construct and the adiabatic experiment we propose to explore it can be used as a measure of the unknown molecule-polaron interaction strength, and leaves open the possibility of observing a novel state; a mixture of molecules and polarons.

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